ZETA MATHS

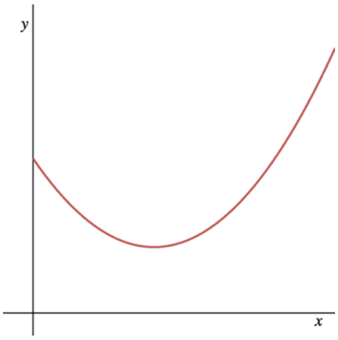
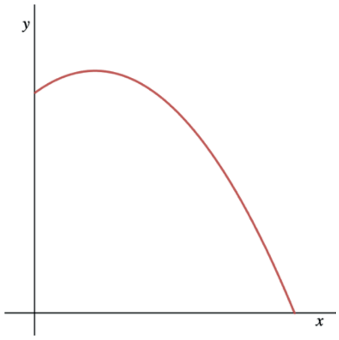
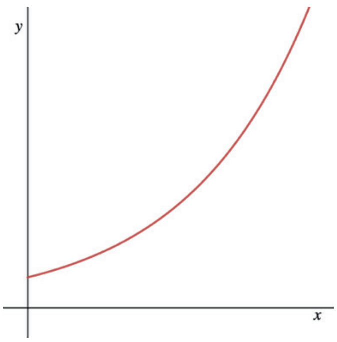
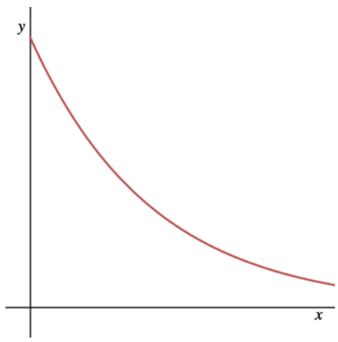
Higher

Applications of Mathematics

Learning Checklist

This checklist covers every skill that learners need for success at Higher Applications of Mathematics. Each section of this checklist corresponds to the **Zeta Maths Higher Applications of Mathematics** textbook (available from shop.zetapress.co.uk or Amazon). The topic names in this document are linked for easy navigation of the checklist and colour coded to correspond with skills: **Mathematical Modelling**, **Statistics and Probability**, **Finance** and **Planning and Decision Making**. Some section numbers are missing as these correspond to revision chapters in the textbook.

Contents

Section	Topic	Skills			
1	**Modelling Situations**				
1.1	Systematic estimation	To find approximate solutions to problems that would be difficult to solve precisely, we make and state **assumptions** about the variables – broad estimates that ignore minor details and special cases – and apply them to **simple mathematical models**.			
1.2	Modelling situations with graphs	When considering graphs that model particular situations, pay attention to: • the **slope** of the graph at different points, which corresponds to the **rate of change** of the output variable • key points on the graph, such as **intercepts** • any points where the **behaviour changes** suddenly or significantly Features of the graph should match up to aspects of the situation being modelled.			
1.3	Linear models	Linear models involve a **constant rate of change**. The formula of a linear model has the form $y = a + bx$: • x is the independent variable • y is the dependent variable • a is the **y-intercept** – the value of x when $y = 0$ • b is the **slope** – the rate of change of y The graph of a linear model is a **straight line**.			
1.4	Quadratic models	The formula of a quadratic model has the form $y = ax^2 + bx + c$, i.e., it involves the independent variable (x) being squared. The graph of a quadratic model is a **parabola**, which has either a minimum (when $a > 0$) or maximum (when $a < 0$) turning point. 			
1.5	Exponential models	An exponential model can be either: • a **growth model** – the rate of change increases as the independent variable increases • a **decay model** – the rate of change decreases as the independent variable increases  The formula of an exponential model has the form $y = ab^x$ or $y = ab^{-x}$: • a represents the initial value of y (when $x = 0$) • b is related to the rate of growth or decay			

Section	Topic	Skills			
1.6	Recurrence models	Recurrence models describe situations where changes occur or are observed at **discrete intervals**. The output at the previous step is taken as the input for the following step. **Worked Example** An enclosure at a zoo initially holds 400 birds. The population will naturally decline by 8% each year. At the end of each year, 30 new birds are introduced to the enclosure. Calculate the number of birds in the enclosure after two years. 1 year: $400 \times 0.92 + 30 = 398$ 2 years: $398 \times 0.92 + 30 = 396$ (rounded to a whole number)			
2	**Units of Measure**				
2.1	Suitable units of measure	The units of an output variable can be determined by combining and simplifying the units of the input variables according to the model. **Worked Example** Given that A is measured in kilograms and B is measured in metres, deduce the units of C where $C = 2A/3B^2$. $\frac{2A}{3B^2} \rightarrow$ kilograms/(metres)^2, so the units of C are "kilograms per square metre" or kg/m^2.			
2.2	Consistency of units in a comparison	When making a **direct comparison** between two quantities, the units of each must be **consistent**, taking into account other factors which must be assumed to be constant to make a fair comparison. **Worked Example** Joey typed an essay in 45 minutes, and Hilda typed an essay in 52 minutes. Explain why we cannot conclude that Joey types more quickly than Hilda. The difference in times may be due to some other factor that differs, such as the length of the essay.			
2.3	Consistency of units in a formula	Units should be consistent both in the way they are defined and in the way they relate to each other: • One unit of a measure must be **exactly the same** as another unit of the same measure. • There must be a **sensible connection** between the units of the input and output variables. **Worked Example** An examination lasts 80 minutes and consists of 10 questions. Students should therefore spend 8 minutes working on each question. Explain why this may be incorrect. The questions may not all be of equal length or difficulty and students may wish to use some of their time to review the exam.			
3	**Error and Tolerance**				
3.1	Absolute and relative errors	The amount by which the true value of a measurement deviates from the observed value is known as the **error**. An **absolute error** is one expressed as a fixed value, and a **relative error** is one expressed as a proportion of the observed value. For example, if the reading of 40 miles per hour (mph) on a speedometer is known to be accurate to within 1.5 mph, i.e., the true speed is known to lie between 38.5 mph and 41.5 mph, then: • the **absolute error** is 1.5 mph; • the **relative error** is $1.5 \div 40 = 3.75\%$. We can write this measurement as 40 ± 1.5 mph or 40 mph $\pm 3.75\%$.			

Section	Topic	Skills			
3.2	Calculating limits for compound measures	The range of values of a dependent variable determined by a formula will depend on the errors of the independent variables. **Worked Example** Use the formula density = mass ÷ volume to determine the minimum and maximum values of density when mass = 10 kg ± 2% and volume = 8 m^3 ± 5%. Density will be minimised when mass is as small as possible and volume is as large as possible: Min. density = (98% of 10) ÷ (105% of 8) = 9.8 ÷ 8.4 = 1.17 kg/m^3 Density will be maximised when mass is as large as possible and volume is as small as possible: Max. density = (102% of 10) ÷ (95% of 8) = 10.2 ÷ 7.6 = 1.34 kg/m^3			
3.3	Estimating errors in a model	When two or more independent variables are multiplied and/or divided in a formula, we can estimate the relative error of the dependent variable by **summing the relative errors** of the independent variables. For example, given that density = mass ÷ volume, if the relative error of mass is 2% and the relative error of volume is 5%, then the estimated relative error of density is 2% + 5% = 7%.			
3.4	Tolerance	When two variables are directly or indirectly proportional to each other, or when there is a linear relationship between them, their relative errors are approximately equal. This can be used to estimate the **tolerance**, or **maximum permitted error**, of the independent variable that is required to ensure that the dependent variable remains within a given range. **Worked Example** Two variables G and H are related by the formula $H = \frac{6}{G}$. The value of H must be 20 with an absolute error of no more than 1.2. Estimate the maximum absolute error of G. Rearrange formula: $G = \frac{6}{H}$ Estimated value of G: $G = 6 \div 20 = 0.3$ Estimated relative error: 1.2 ÷ 20 = 6% Estimated absolute error: 6% of 0.3 = 0.018			
3.5	Accuracy and precision	The **accuracy** of a measurement refers to the correctness of the value when compared with the true value. The **precision** of a measurement refers to the exactness of its value, indicated by the number of significant figures or the relative error. For example, say it takes a runner 32 minutes to complete a race. An estimate of 30 minutes would be **accurate**, since it is correct to the nearest 10 minutes, but **imprecise** since it is given to one significant figure (relative error of ~16.7%). An estimate of 33.2 minutes would be **inaccurate**, since it is not correct even when taking rounding into account, but **precise** since it is given to three significant figures (relative error of ~0.15%).			
4	**Evaluating and Interpreting Models**				
4.1	Interpreting models	Mathematical models may be presented in a range of ways, including **formulas**, **graphs** and **tables**. From each of these representations we can identify the type of model (linear, quadratic, exponential or recurrence), interpret key features of the model and make predictions based on the model.			

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4.2	Evaluating the output of models	In evaluating whether the output of a model is reasonable, we should consider: • whether the **units of measure** are consistent • whether the claimed **precision** of the output is justified • whether the model has been applied beyond the limits of its validity (**extrapolation**), perhaps resulting in implausible or impossible claims			
4.3	Improving models	When models produce inaccurate or implausible outputs, we may make minor modification to improve its reliability. This may involve **adjusting the parameters** of the model or **defining limitations** on the inputs of the model. In some cases, a **different type of model** may be more appropriate.			
5	**Using Software for Modelling**				
5.1	Using basic spreadsheet functions	Functions in spreadsheets must be preceded by the '=' symbol. As well as using the arithmetic operators +, –, * (multiply), / (divide) and ^ ("to the power of") to perform calculations, we should be familiar with the following functions: • ROUND (round to specified number of decimal places) • INT (round *down* to the nearest integer) • ABS (gives the value without its sign) • SUM (adds all values in a range) • PRODUCT (multiplies all values in a range) • AVERAGE (finds the mean of a range of values) • MIN (finds the minimum value in a range) • MAX (finds the maximum value in a range) • MEDIAN (finds the median of a range of values) • STDEV (finds the standard deviation of a range of values) We should also be familiar with the following functions involving **conditional statements**: • AND (returns TRUE if *both* statements are true) • OR (returns TRUE if *at least one* statement is true) • IF (returns specified output depending on whether statement is true) • COUNTIF (returns the number of values in a range satisfying the conditional statement)			
5.2	Setting up and using key variables	When a variable is used in multiple formulae in different parts of a spreadsheet, we should set it up as a **key variable** which can then be referenced, i.e., type it into one cell and then reference that cell in the formulae. We usually set the **parameters** of a mathematical model up as key variables. When making reference to a key variable, we should use an **absolute reference**, e.g., E6.			
5.3	Using Goal Seek	The **Goal Seek** function is used to determine the value of a parameter or input that will give the desired output for a model. This allows us to "work backwards" in a way that would be extremely mathematically complex. It is essentially an automated and very precise trial-and-error process. The Goal Seek function can be found in the 'Data' tab under the 'What-if Analysis' menu. It may be necessary to manually adjust the value produced to within a justified level of precision while still ensuring it gives a valid solution.			
5.4	Implementing models	Models which involve **iteration**, where a process is repeated many times, are most effectively implemented using a spreadsheet. The output from the previous application of the process can be used as input for the next. The spreadsheet can carry out a large number of calculations very quickly, whereas it would take a long time using a calculator. This is especially useful for **recurrence** models, but iteration can also be used to approximate continuous models.			

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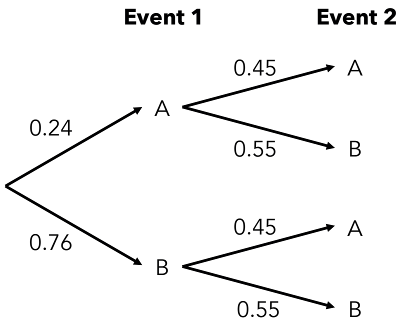
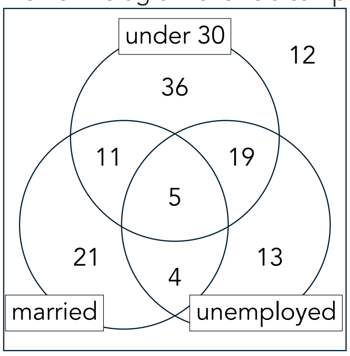
5.5	Generating graphs and charts	We can use spreadsheet software to generate appropriate graphical displays in order to **visualise** the numerical output of a model and form a **subjective impression** of the output. Graphs can be generated by highlighting the data and selecting the appropriate display from the 'Insert' tab. We should then ensure the chart has an appropriate title and axes that are labelled. We should also take into account the readability of graphs and charts, especially if they will be rendered in grayscale or black-and-white.			
7	**Probability**				
7.1	Tree diagrams	The 'branches' of a tree diagram are labelled with the probability of the outcomes they lead to, and we multiply along the branches to determine the probabilities of the respective combined outcomes. **Worked Example**  Use the tree diagram above to determine the probability of outcome B occurring in **exactly one** event. P(AB) + P(BA) = (0.24 × 0.55) + (0.76 × 0.45) = 0.132 + 0.342 = 0.474			
7.2	Venn diagrams	In a Venn diagram, the number in each region is the number of elements of the data set that satisfy the corresponding criteria. We should be able to both construct and interpret a Venn diagram. **Worked Example** The Venn diagram shows a sample of the results of a survey.  One person from the survey is selected at random. Determine the probability that they are married but not unemployed. There are 121 people in the survey altogether. 32 people (21 + 11) in the sample are married but not unemployed. So the probability is $\frac{32}{121}$.			
8	**Statistical Literacy**				
8.1	Type of data	**Numerical** data represents a quantity, expressed in numbers, that could be used to carry out calculations. It is either: • **discrete** (countable), e.g., number of students, or • **continuous** (measured), e.g., temperature **Categorical** data represents a quality, usually expressed in natural language. It is either: • **ordinal** (can be ordered), e.g., "mild, hot, very hot", or • **nominal** (used to label), e.g., eye colour			

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8.2	Populations and samples	When gathering data, the **population** is the potential pool of all sources of data meeting the criteria, and a **sample** is a subset of that population. We should aim for samples to be **representative** of the population, taking into account different characteristics of the members of the population that may impact on the data being gathered.			
8.3	Outliers	An outlier is a data value that is out of keeping with the other values in the data set. It may be significantly greater or less than other values or it may represent an improbable or impossible outcome. If they are the result of **human error**, it may be appropriate to remove them from the data set before carrying out analysis. If they represent accurate but **genuinely unexpected outcomes**, they should not be removed but should be acknowledged.			
8.4	Data gathering and bias	When checking a sample for bias, consider whether it is **random**, and whether it is **representative**. Other factors should also be taken into account, such as the accuracy of self-reported data and the reliability of the source.			
9	**Statistical Diagrams**				
9.2	Tables	We use tables to present the total or proportions of **categorical data**. The following RStudio commands can be used to produce tables, although we should ensure they are properly formatted with appropriate titles and labels: • `table(X)` – produces a frequency table for the variable X • `table(X,Y)` – produces a contingency table for the variables X and Y • `prop.table(table(X))` – produces a table of the proportions of each observation for the variable X			
9.3	Bar charts and pie charts	Bar charts and pie charts are used to visualise **categorical data**. Bar charts are useful for comparing totals, and pie charts are useful for comparing proportions. We use the following RStudio commands: • `barplot(table(X))` – produces a bar chart for the variable X • `pie(table(X))` – produces a pie chart for the variable X We should include arguments to ensure our diagrams have **appropriate titles and labels**.			
9.4	Boxplots	Boxplots visualise the **five-figure summary** and **interquartile range** of a **numerical** data set. We use the following RStudio commands: • `boxplot(X)` – produces a boxplot for the variable X • `boxplot(X,Y)` – produces a comparative boxplot for the variables X and Y We should include arguments to ensure our diagrams have **appropriate titles and labels**.			
9.5	Histograms	Histograms visualise the **distribution** of a **continuous numerical** variable. We use the following RStudio command: • `hist(X)` – produces a histogram for the variable X We should include arguments to ensure our diagrams have **appropriate titles and labels**.			

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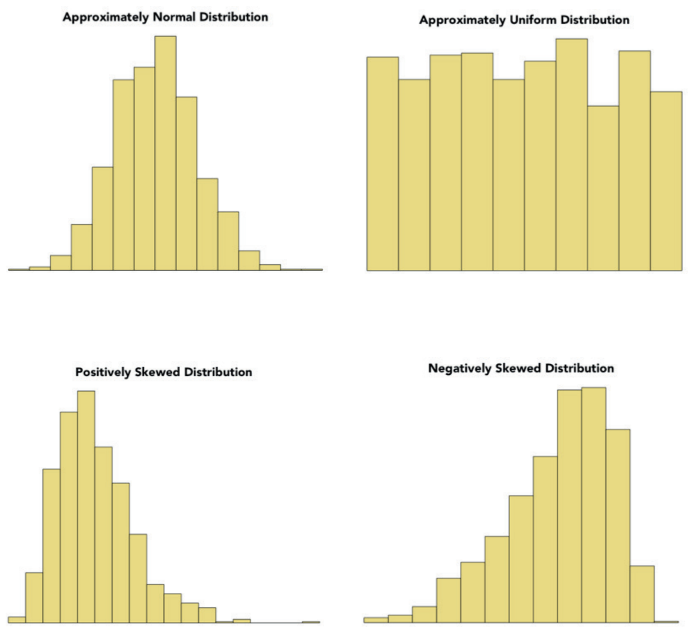
9.6	Scatterplots	Scatterplots visualise **bivariate data** in order to visualise what relationship exists, if any, between the two variables. They can be useful in identifying the **type of model** that might best represent the relationship. We use the following RStudio command: • `plot(X,Y)` – produces a scatterplot of Y on X, where `X` is the independent variable and `Y` is the dependent variable We should include arguments to ensure our diagrams have **appropriate titles and labels**.			
9.7	Misleading graphs	Diagrams are sometimes manipulated or produced in a way that they may give a false or misleading impression of what the data shows. This might be achieved by: • adjusting the scale on an axis • selecting or omitting certain data • inappropriate use or comparison of proportions • use of multiple scales			
10	**Descriptive Statistics**				
10.1	Categorical data	For categorical data, the relevant descriptive statistics are the **sample size** and the **proportions**, which can be determined by creating tables. We should make comparisons about the proportions of the **same observation from different samples**. For example, we might compare the proportions of single people from two surveys taken in different towns, rather than compare the proportions of single and married people from the same survey.			
10.2	Distribution of data	There are four types of distribution we should be able to identify from a histogram: 			

<table>
<tr><th>Section</th><th>Topic</th><th>Skills</th><th></th><th></th><th></th></tr>
<tr><td>10.3</td><td>Measures of location and dispersion</td><td>For numerical data sets, a measure of location is a central or "typical" value, and a measure of dispersion gives us information about the variability in the data.

Appropriate measures to use are determined by the distribution of the data:
<table>
<tr><th>Distribution</th><th>Measure of location</th><th>Measure of dispersion</th></tr>
<tr><td>Normal</td><td>Mean</td><td>Standard deviation</td></tr>
<tr><td>Skewed</td><td>Median</td><td>Interquartile range</td></tr>
</table>
We use the following RStudio commands, for the variable X:
• <code>mean(X)</code>
• <code>sd(X)</code> (standard deviation)
• <code>median(X)</code>
• <code>IQR(X)</code> (interquartile range)</td><td></td><td></td><td></td></tr>
<tr><td>10.4</td><td>Comparing data sets</td><td>Comments should show an understanding that the measure of location is an average and the measure of dispersion gives information about the variability of the data. Comparisons should always refer to the context of the data.</td><td></td><td></td><td></td></tr>
<tr><td>11</td><td colspan="5">Correlation and Regression</td></tr>
<tr><td>11.1</td><td>Interpreting scatterplots</td><td>If a scatterplot shows evidence of a linear relationship, the interpretation should comment on the strength and direction of the relationship.

Strength: the correlation may be described as weak if the points are quite dispersed or strong if they are close together. The use of modifiers such as "fairly" is acceptable.

Direction: the correlation may be described as positive (as one variable increases, so does the other) or negative (as one variable increases, the other decreases).</td><td></td><td></td><td></td></tr>
<tr><td>11.2</td><td>Correlation coefficient</td><td>We can run a correlation test in RStudio using the command <code>cor.text(X,Y)</code> to determine the Pearson's product-moment correlation coefficient. The data must satisfty the necessary assumptions, and the test must return a p-value less than 0.05 for the coefficient to be valid.

Whether the coefficient is positive or negative indicates the direction of the relationship, and the strength of the relationship is indicated by the number itself – values close to 0 show a weak relationship, while those close to –1 or 1 show a strong relationship.</td><td></td><td></td><td></td></tr>
<tr><td>11.3</td><td>Fitting a linear regression model</td><td>To fit a linear regression line to model the relationship of Y on X, use the RStudio command <code>lm(Y~X)</code>. This will return as output the y-intercept and slope of the line.

The intercept is the value of the dependent variable when the independent variable is 0. The slope is the rate of change of the dependent variable.

We can add the regression line to an a scatterplot in RStudio using the command <code>abline(lm(Y~X))</code>.</td><td></td><td></td><td></td></tr>
<tr><td>11.4</td><td>Making predictions</td><td>We can use a regression model to make predictions using the RStudio command: <code>predict(lm(Y~X),newdata=data.frame(X=C),interval="pred")</code>
This predicts the value of <code>Y</code> when <code>X=C</code>, so we replace <code>C</code> with the desired input value.

This command produces a predicted value, labelled <code>fit</code>, and the lower (<code>lwr</code>) and upper (<code>upr</code>) bounds of a 95% confidence interval.</td><td></td><td></td><td></td></tr>
</table>

Section	Topic	Skills			
12	**Data Analysis**				
12.1	Hypothesis testing	For a closed research question, the **null hypothesis** (H_0) is that the answer is no, and the alternative hypothesis (H_a or H_1) is that the answer is yes. **Worked Example** Research question: *Is there a correlation between the mass of a car and its braking distance?* State the null and alternative hypotheses. H_0: There is no correlation between the mass of a car and its braking distance. H_1: There is a correlation between the mass of a car and its braking distance.			
12.2	Errors in statistical testing	After carrying out an appropriate statistical test, we either **reject** or **fail to reject** the null hypothesis. There are two types of errors we should be aware of: Type I: Rejecting the null hypothesis when it is true. Type II: Failing to reject the null hypothesis when it is false.			
12.3	Confounding variables	A confounding variable is an external factor that affects both variables being studied and may make it difficult to determine the true relationship between them. For example, a study may find that people who consume large amounts of coffee are more likely to have heart disease, but high levels of stress may cause someone to drink more coffee and raise their risk of heart disease. So stress may be a confounding variable in this scenario.			
12.4	Confidence intervals and *p*-values	A **confidence interval** is the range of values within which the true value of a statistical parameter is likely to lie. For the statistical tests in this course, we are interested in whether the 95% confidence interval contains 0. The ***p*-value** is the probability that the observed data could have come about by random chance if the null hypothesis was true. A value of less than 0.05 supports the rejection of the null hypothesis. **Worked Example** Research question: *Is there a difference in the proportions of single people between Town A and Town B.* An appropriate statistical test produced a confidence interval of (–0.15, 0.08) and a *p*-value of 0.062. Interpret these results in context. There is a 95% probability that the difference in proportions lies between –0.15 and 0.08. Since this interval contains 0, there is evidence to suggest that there is no significant difference in the proportions of single people between the two towns. Since $p > 0.05$, we fail to reject the null hypotheses – there is evidence to suggest that there is no significant different in the proportions of single people between the two towns.			
12.5	Independent *t*-tests	An **independent *t*-test** is used to compare the **means** of two independent samples. It assumes data is randomly sampled, continuous and normally distributed. The associated research question has the form, "Is there a difference in the mean value of the variable between Population A and Population B?" We use the RStudio command `t.text(X,Y)`, where X and Y are the two samples being compared.			

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12.6	Paired *t*-tests	A **paired *t*-test** is used to compare **two sets of observations for the same subject**. Rather than comparing the mean of two variables, it compares the mean of the differences between the two with 0. It assumes data is randomly sampled, continuous and normally distributed. The associated research question has the form, "Is there a difference between Observation A and Observation B?" The two observations may be "before" and "after", for example. We use the RStudio command `t.text(X,Y,paired=TRUE)`, where X and Y are the two samples being compared.			
12.7	*z*-tests for two proportions	A ***z*-test for two proportions** is used to compare the **proportions** of a variable between two independent samples. It assumes the data is randomly sampled and categorical. The associated research question has the form, "Is there a difference in the proportions of the variable between Population A and Population B?" We use the RStudio command `prop.test(x=c(a,b),n=c(n1,n2))`. This compares the proportions `a/n1` and `b/n2`.			
14	**Financial Products**				
14.1	Savings products	In choosing the most appropriate savings product, factors that should be considered include **interest rate**, **restrictions** on transactions, and **incentive schemes**. Products with more restrictions tend to have higher interest rates, whereas those offering more flexibility will have lower interest rates.			
14.2	Credit cards and loans	**Credit cards** allow for the purchase of goods and services with deferred payment. Using a credit card offers extra protection to the customer. A minimum repayment must be made each month, but paying only this may result in additional interest charges. **Loans** are amounts of money that are borrowed for an agreed period of time. They are usually paid back in small installments over time, but may also be repaid in full or on an "interest-only" basis.			
14.3	Insurance	Insurance products are a form of **financial protection** against unexpected and/or very large payments. The customer makes relatively small payments (sometimes called a **premium**), either as a one-off purchase or on an ongoing basis, and the insurer agrees to pay a **benefit** when the conditions of the policy are met. Some policies include an **excess**, an amount payable by the customer towards the expense before the benefit is received.			
15	**Effective Rates of Interest**				
15.1	Compounding interest	**Worked Example** Trish deposits £1500 into a savings account with an effective rate of interest of 3.7% per year. Calculate the value of the deposit after 2.5 years. Multiplier: 100% + 3.7% = 103.7% = 1.037 After 2.5 years: $1500 \times 1.037^{2.5} = £1642.62$			
15.2	Different time frequencies	**Worked Example** Paul takes out a loan for £6250 with an effective rate of interest of 5.5% per annum. Calculate the accumulated value of the loan after 20 months, assuming no repayments are made. Multiplier: 100% + 5.5% = 105.5% = 1.055 Since the rate is annual (12 months) and the term is 20 months, the exponent is $\frac{20}{12}$. After 20 months: $6250 \times 1.055^{\frac{20}{12}} = £6833.36$			

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15.3	Converting between time frequencies	**Worked Example** The effective rate of interest of a financial product is 6.1% per year. Calculate the effective rate of interest per month. Multiplier for 1 year: $100\% + 6.1\% = 106.1\% = 1.061$ Multiplier for 1 month: $1.061^{\frac{1}{12}} = 1.00495$ Rate for 1 month: $1.00495 - 1 = 0.00495 = 0.495\%$ per month			
15.4	Combining interest rates	**Worked Example** The effective rate of interest for a savings account in 2024 was: • 2.5% per year from 1st January to 30th June • 0.5% per month from 1st July to 30th September • 2.8% per year from 1st October to 31st December Calculate the effective rate of interest for the entire year. 1st Jan to 30th Jun (6 months): $1.025^{\frac{6}{12}}$ 1st Jul to 30th Sep (3 months): 1.005^3 1st Oct to 31st Dec (3 months): $1.028^{\frac{3}{12}}$ Effective annual rate: $1.025^{\frac{6}{12}} \times 1.005^3 \times 1.028^{\frac{3}{12}} = 1.0348$ $1.0348 - 1 = 0.0348 = 3.48\%$ per year			
16	**Accumulation**				
16.1	Irregular repayments	There are two methods for accumulating the balance of an account: **Method 1 – chronological** Accumulate for the time between transactions, tracking the value of the balance over the specified time period. **Method 2 – by payment** Accumulate each transaction individually and sum the total. For irregular payments, it's important to pay close attention to the time period for which interest is accumulated.			
16.2	Regular fixed payments	Making payments of the same value at regular intervals will produce repetition and/or predictable patterns in the accumulation calculations. Either method (chronological or by payment) may be used.			
16.3	Regular fixed payments (spreadsheet)	**Step 1**: Calculate the effective rate of interest for the time period between payments (usually monthly). **Step 2**: Complete the initial setup of the payment schedule with the initial balance, the regular payment amount and the balance before interest. **Step 3**: Calculate the accumulated balance after the first time interest is applied (remember to round values to 2 decimal places). **Step 4**: Complete the payment schedule for the specified time period to find the balance after the final payment but before interest is applied. **Step 5**: Calculate the final balance by applying interest one final time.			
16.4	Regular variable payments	This is similar to making regular fixed payments, but the payment amount will change by a fixed amount or proportion. For example, a person may increase the amount they deposit into a savings account each month to build up a good savings habit.			
16.5	Regular variable payments (spreadsheet)	This is similar to making regular fixed payments, but making sure to adjust the formula for the payment amount so that it changes over time.			
16.6	Variations (spreadsheet)	We sometimes must make adjustments to the formulae in a spreadsheet to take account of variations such as unexpected transactions or changes to interest rates or the regular payment amount. It's important to ensure the change is made at the correct point in the payment schedule.			

<table>
<tr><th>Section</th><th>Topic</th><th>Skills</th><th></th><th></th><th></th></tr>
<tr><td>17</td><td colspan="5">Present Value</td></tr>
<tr><td>17.1</td><td>Fixed interest rate</td><td>Worked Example
The effective rate of interest of a savings account is 4.2% per annum. Calculate the present value of £2100 in 18 months.

If the present value is £P, then its value after 18 months would be:
$P \times 1.042^{\frac{18}{12}}$
Set this equal to our goal: $P \times 1.042^{\frac{18}{12}} = £2100$
Which gives: $P = 2100 \div 1.042^{\frac{18}{12}} = £1974.32$</td><td></td><td></td><td></td></tr>
<tr><td>17.2</td><td>Variable interest rate</td><td>Worked Example
Tracey deposits a lump sum into her savings account on 1st January 2024. The effective rate of interest on the account is:
• 3.4% per annum from 1st January 2024 to 31st March 2024
• 0.4% per month from 31st March 2024 onwards
On 30th June 2024, the balance in the account is £1670.51. Calculate how much Tracey originally deposited.

If the present value is £P, then its value on 30th June 2024 would be:
$P \times 1.034^{\frac{3}{12}} \times 1.004^3$
Set this equal to our goal: $P \times 1.034^{\frac{3}{12}} \times 1.004^3 = £1683.90$
Which gives: $P = 1683.90 \div (1.034^{\frac{3}{12}} \times 1.004^3) = £1650.00$</td><td></td><td></td><td></td></tr>
<tr><td>17.3</td><td>Using Goal Seek</td><td>The Goal Seek function can be used to determine the regular payment amount required to meet a particular goal in a set amount of time.</td><td></td><td></td><td></td></tr>
<tr><td>18</td><td colspan="5">Loan Repayments</td></tr>
<tr><td>18.1</td><td>Loan calculations</td><td>Worked Example
A loan for £8450 is taken out over 60 months. The effective rate of interest on the loan is 3.8% per annum.
Calculate the total amount to be repaid if:
(a) the loan is repaid at the end of the term;
(b) interest-only repayments are made each month.

(a) The accumulated value of the loan is: $8450 \times 1.038^5 = £10{,}182.24$

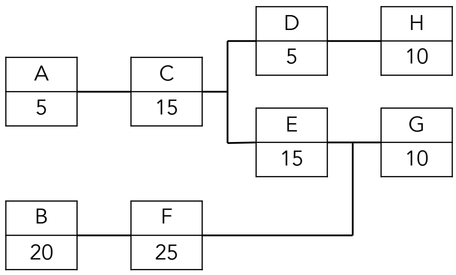
(b) The monthly effective rate of interest is $1.038^{\frac{1}{12}} - 1 = 0.311\%$
Monthly repayment amount: 0.311% of 8450 = £26.28
Total repayment amount: $60 \times 26.28 + 8450 = £10{,}026.80$</td><td></td><td></td><td></td></tr>
<tr><td>18.2</td><td>Repayment schedules</td><td>When loans are repaid by regular payments, a repayment schedule can be used to show the interest and capital content of each payment. Interest content is calculated on the loan outstanding from the row above and the capital content is the difference between the payment amount and the interest content. The capital content is deducted from the loan outstanding.

In order to bring the loan outstanding to exactly £0 at the end of the term of the loan, the final repayment may be slightly more or less than the regular repayment amount.

An example of a partial schedule is shown below:

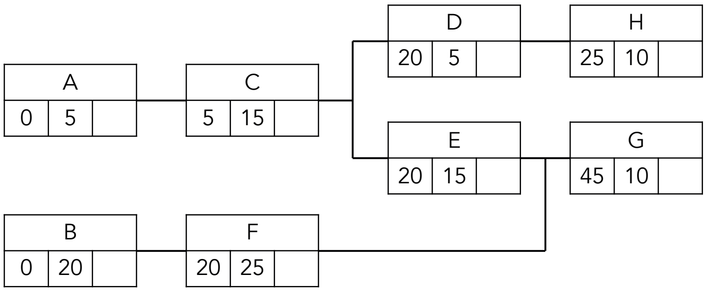
<table>
<tr><th>Month</th><th>Repayment</th><th>Interest content</th><th>Capital content</th><th>Loan outstanding</th></tr>
<tr><td>0</td><td></td><td></td><td></td><td>£8450</td></tr>
<tr><td>1</td><td>£154.61</td><td>£26.30</td><td>£128.31</td><td>£8321.69</td></tr>
<tr><td>2</td><td>£154.61</td><td>£25.90</td><td>£128.71</td><td>£8192.98</td></tr>
</table></td><td></td><td></td><td></td></tr>
</table>

Section	Topic	Skills			
18.3	Repayment schedules (spreadsheet)	The setup of a loan repayment schedule using a spreadsheet is the same as the hand-written version above. Interest rates and repayment amounts are set up as key variables, and the Goal Seek function may be used to determine the repayment amount or interest rate.			
19	**Financial Planning**				
19.1	Gross income	Gross income is the **total earnings** from all sources. It may be comprised of basic pay, overtime, commission and bonuses. It is most commonly calculated on a weekly, four-weekly, monthly or annual basis.			
19.2	National Insurance	National Insurance is a **deduction** calculated based on gross income, paid by almost all working people. National Insurance contributions must be paid to qualify for certain benefits and the State Pension. Rates are set by the UK government and may vary from year to year. Rates will be provided in the Data Booklet if they are required in the exam.			
19.3	Income tax	Income tax is usually the **largest deduction** from gross income. It is a progressive tax, which means that the rate is higher for higher earners. 'Taxable income' may not always be the same as gross income, as some deductions are 'pre-tax'. Income tax rates and bands are set by the Scottish government each year. Rates will be provided in the Data booklet if they are required in the exam.			
19.4	Net income	**Net income** is the difference between gross income and total deductions. As well as National Insurance and income tax, other deductions might include **workplace pensions**, **student loan repayments**, fees for **professional bodies** and **child maintenance**. Pay close attention to whether a deduction is calculated on gross income or taxable income, and whether it is deducted pre-tax. For example, pension contributions are calculated on gross income and deducted pre-tax.			
19.5	Other taxes	Other taxes that are not usually deducted from gross income include **Land and Buildings Transaction Tax** (LBTT), **Value Added Tax** (VAT), **Council Tax** and **Vehicle Tax**. We may have to use information about these from the Data Booklet in the exam.			
19.6	Pensions	For a **defined contribution** pension scheme, we can use a spreadsheet to estimate the contribution required to provide a certain level of income during retirement. The first worksheet is used to calculate the total of pension contributions required by retirement age to provide the desired income, and the second worksheet is used to determine the contribution amount required to achieve that.			

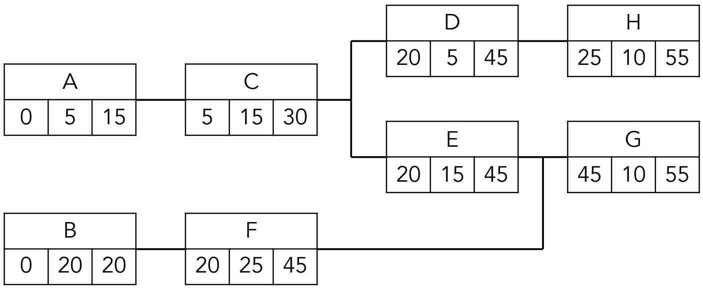
Section	Topic	Skills			
19.7	Inflation and purchasing power	Inflation is a general increase in the price of goods and services over time, usually measured against an **inflation index** such as the CPI or RPI. Inflation results in a decrease in the **purchasing power** of money over time – fewer goods and services can be purchased than previously with the same amount of money. **Worked Example** The CPI was 98.5 in 2013 and 130.5 in 2023. Calculate the amount of money in 2023 that would have the equivalent purchasing power as £350 in 2013. Relative rate of inflation: (130.5 – 98.5) ÷ 98.5 = 0.3248… Increase £350 by relative rate: 350 × (1 + 0.3248…) = £463.705… Therefore £350 in 2013 had the same purchasing power as £463.71 in 2023.			
19.8	Financial planning risks	Decisions about financial planning usually involve a certain amount of risk, whether related to personal circumstances, the type of financial product or the economy more widely. We should be able to identify and discuss potential risks involved in different scenarios.			
21	**Project Planning**				
21.1	Activity networks and critical paths	An **activity network** is a way to represent a compound project, with tasks represented by **nodes**, connected according to their precedence relations. The **critical path** of a network is the sequences of activities which determines the minimum completion time of the project. **Worked Example** For the activity network shown, durations are in days. Determine the critical path and the minimum time to complete the project.  The critical path must start with an activity with no predecessor (A or B) and end with an activity with no successor (G or H). Possible paths are: • A-C-D-H (35 days) • A-C-E-G (45 days) • B-F-G (55 days) We select the path with the longest duration. Therefore the critical path is B-F-G and the minimum completion time is 55 days.			

<table>
<tr><th>Section</th><th>Topic</th><th>Skills</th><th></th><th></th><th></th></tr>
<tr><td>21.2</td><td>Constructing PERT charts</td><td>
In a Programme Evaluation and Review Technique (PERT) chart, each activity is represented by a node with its earliest start time, duration and latest end time shown.

The earliest start time is determined by a forward scan, using the earliest start time and duration of predecessors:

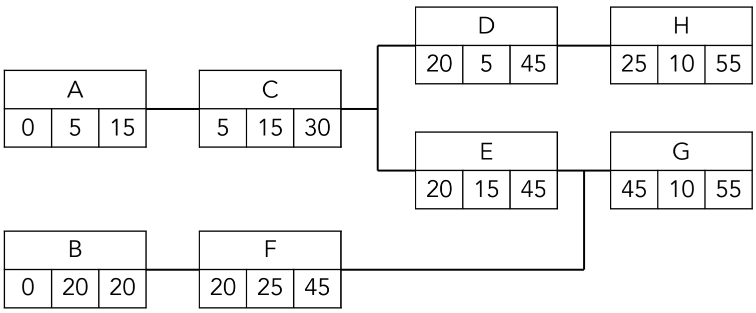


The latest end time is determined by a backward scan, using the latest end time and duration of successors:



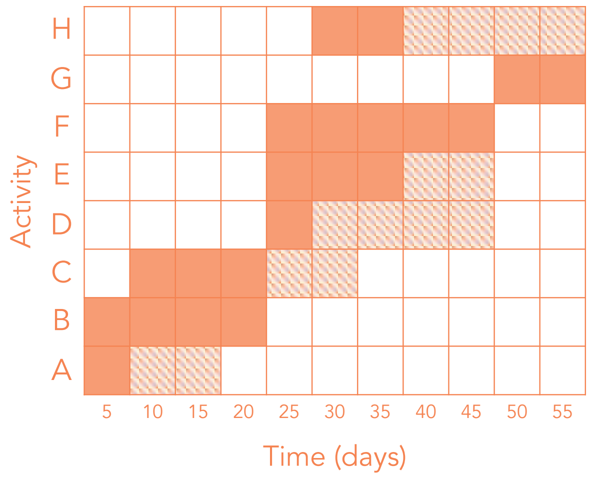
</td><td></td><td></td><td></td></tr>
<tr><td>21.3</td><td>Interpreting PERT charts</td><td>
The float (or slack) time of an activity is the amount of time by which it may be delayed without impacting on the overall completion time of the project.

Worked Example
A completed PERT chart is shown below, with times given in days.



Determine the impact on the completion time of the project if Actviity C is delayed by 20 days.

The float time for activity C is (30 – 5) – 15 = 10 days.
Therefore, a delay of 20 days to Activity C would delay the project by 20 – 10 = 10 days overall (the new minimum completion time would be 65 days).
</td><td></td><td></td><td></td></tr>
</table>

Section	Topic	Skills			
21.4	Constructing Gantt charts	In a **Gantt chart**, the horizontal axis is a timeline and activities are represented by bars. **Worked Example** Use the information from the PERT chart above to construct a Gantt chart.  Different shading is used to represent the float time of activities.			
21.5	Interpreting Gantt charts	Similarly to a PERT chart, a Gantt chart can be used to determine the critical path, calculate float time and interpret the impact of changes to activities. If float times are not shown on the chart, additional information is usually required to interpret the precedence order of all activities.			
22	**Expected Value**				
22.1	Calculating expected value	The **expected value** is the mean value of all possible outcomes, weighted by their probability. **Worked Example** In a game, the player rolls a six-sided dice. They score: • 0 points for rolling a 1; • 2 points for rolling a 2, 3, 4 or 5; • 10 points for rolling a 6. Calculate the expected number of points scored from each roll. Expected value = $\left(\frac{1}{6} \times 0\right) + \left(\frac{4}{6} \times 2\right) + \left(\frac{1}{6} \times 10\right) = 3$ points			
22.2	Introducing control measures	A **control measure** is an action taken to alter the probability of certain outcomes in order to minimise expected cost and/or maximise expected benefit. A control measure is usually considered effective if its actual cost is less than the amount by which it reduces the expected cost. **Worked Example** The probability of a company incurring a fine of £2000 is 0.15. If they introduce a control measure at a cost of £120, the probability will decrease to 0.08. Determine whether the control measure is effective. Expected cost without control measure: $0.15 \times 2000 = £300$ Expected cost with control measure: $120 + 0.08 \times 2000 = £280$ Since the expected cost is less *with* the control measure, it is effective.			